The eigenvalues of the open-loop system are (− 2.8166, 1.0276). Thus, the short-period mode has become two real poles since the c.g. is so far aft.

To apply our results to this system, we need to transform it to the controllable form. This can be done using the transformation $A_c = T^{-1}AT$ and $B_c = T^{-1}B$ where

$$A_c = \begin{bmatrix} 0 & 1 \\ 2.8944 & -1.7889 \end{bmatrix}, \qquad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{22}$$

and

$$T = \begin{bmatrix} -0.1548 & -0.0021 \\ -0.181 & -0.1692 \end{bmatrix} \tag{23}$$

The objective of the design is to locate the closed-loop poles at $(-2 \pm j1.9)$. These poles are suitable in terms of flying qualities.[6]

Equations (18) and (19) result in the state-weighting matrix

$$Q = \begin{bmatrix} 49.5347 & 0 \\ 0 & -8.2090 \end{bmatrix} \tag{24}$$

The solution of Eq. (5) is given by

$$P = W_1 W_2^{-1} \tag{25}$$

where $W = [W_{1T} W_{2T}]^T$ is the matrix of the eigenvectors corresponding to the stable eigenvalues of the Hamiltonian matrix

$$H = \begin{bmatrix} A_c & -B_c B_c^T \\ -Q & -A_c^T \end{bmatrix} \tag{26}$$

Therefore, we may compute

$$P = \begin{bmatrix} 35.697 & 10.5322 \\ 10.5322 & 2.2216 \end{bmatrix} \tag{27}$$

The required optimal LQ gain is given by

$$K = [10.5322 \quad 2.2216] \tag{28}$$

The optimal controller gain for the original system is given by

$$KT^{-1} = [-53.4699 \quad -12.4609] \tag{29}$$

and the state-weighting matrix for the original system is given by

$$T^{-T}QT^{-1} = \begin{bmatrix} 1725.4 & 0.3187 \\ 318.7 & -295 \end{bmatrix} \tag{30}$$

Note that, even for this simple design that represents a common sort of problem, a nonpositive semidefinite state weighting is needed to achieve the military specifications.

V. Conclusions

We have developed a relation for finding the state-weighting matrix Q in the linear quadratic performance index to yield specified closed-loop poles for single-input systems. This relation expresses Q in terms of the open-loop and closed-loop eigenvalues. The relation shows the need for nonpositive semidefinite state weighting for some selections of the closed-loop poles. This means that restricting our attention to a positive semidefinite state-weighting matrix does not allow arbitrary placement of the closed-loop poles. To show that nonpositive semidefinite Q can be needed in practical situations, we designed a longitudinal stability augmentation system where such a Q was needed to conform to the MIL-specified closed-loop poles.

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Improved Tracking of an Agile Target

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I. Introduction

As the term is commonly interpreted, tracking refers to estimating the current state of the target; e.g., position, velocity, etc., from a spatiotemporal observation.[1] Prediction of future target location can be made on the basis of this estimate. Even with the new generation of electro-optical (EO) sensors, tracking an agile target is a difficult task. While there is well-developed theory of linear (or essentially linear) estimation and prediction, quite useful in benign environments, it is difficult to adapt the results to situations involving rapid changes. The models upon which the algorithms are based frequently do not exhibit the discrete maneuver regimes typical of a hostile encounter.

Synthesis of model-based tracking algorithm begins with the equations of motion of the target. Motion dynamics are conventionally rendered in terms of a linear Gauss Markov (LGM) model

$$dx_t = Ax_t \, dt + dw_t \tag{1}$$

in which x_t is the state process, including the position, velocity, and in some instances, acceleration of the target, and $(d/dt)w_t$ is a wideband (white) process of intensity W [$W dt = (dw_t)(dw_t)'$], selected to introduce uncertainty into the path of the target. It is traditionally assumed that the tracker avails itself of sensors that measure target motions,

$$dy_t = Dx_t \, dt + dn_x \tag{2}$$

Received Jan. 9, 1991; revision received April 19, 1991; accepted for publication May 21, 1991.

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where n_x is a Brownian motion process with intensity $R_x > 0$, independent of both w_t and the initial conditions on Eq. (1). In applications, both Eqs. (1) and (2) are frequently localizations of a nonlinear relations; e.g., range and bearing measurements, which more comprehensively describe the evolution of the encounter.

The solution to the mean-square estimation problem formed from Eqs. (1) and (2) can be phrased as follows. Denote the information pattern—filtration—generated by y_t by Y_t. The conditional mean of the local encounter state $[\hat{x}_t = E(x_t/Y_t)]$ is given by the (extended) Kalman filter (EKF)[2]:

$$d\hat{x}_t = A\hat{x}_t\, dt + P_{xx}D'R_x^{-1}\, d\nu_x \tag{3}$$

where $d\nu_x = dy - D\hat{x}\, dt$ (the innovations), and P_{xx} is the error covariance matrix.

Note that when P_{xx} is small, the innovations process receives little note, and the estimate propagates forward along the field of the unexcited system. As the uncertainty in the state estimate increases, new information is accorded increasing value; i.e., as the estimator becomes less sure of the true state, it is more willing to modify its prior estimate in response to new data. The matrix P_{xx} is given by the solution to a matrix differential equation:

$$dP_{xx} = [AP_{xx} + (AP_{xx})' + W - P_{xx}D'R_x^{-1}DP_{xx}]\, dt \tag{4}$$

The right-hand side of Eq. (4) is composed of readily interpreted terms. The error variance is amplified by the system dynamics $[(AP_{xx} + P_{xx}A')dt]$ and is reduced by observation process $(P_{xx}D'R_x^{-1}\, DP_{xx}\, dt)$. The equation of evolution of the error variance process P_{xx} is responsive to the influence of the exogenous disturbance as well. As the intensity of this excitation W increases, the increment in P_{xx} is increased proportionately. From Eq. (1), W has another interpretation. It is the intensity of the Y_t-predictable quadratic variation of x_t; i.e.,

$$W\, dt = d\langle x,x\rangle_t \tag{5}$$

The EKF algorithm is quite adept at following motions in undemanding tracking environments such as occur at long ranges. Target motions are small relative to the field of view (FOV), and any reasonable tracking algorithm will maintain the target in the tracking window. At shorter ranges the problem becomes more complex. Perceived target motions become more volatile, and maneuver occurrences should be incorporated into the motion model. A conscientious representation would differentiate between distinguishable constituents of the acceleration process. On time scales of interest, the acceleration has a natural decomposition into continuous and discontinuous parts. The former is produced as a combination of a variety of parasitic effects, individually small, but non-negligible in aggregate, and is well modeled by an LGM process. The latter is produced by operator-induced motions. As a pilot seeks to avoid being tracked, and to make his future location difficult to predict, he may select a timed sequence of maneuver accelerations to create a complex motion pattern. The LGM algorithm, Eqs. (3) and (4), finds such motions difficult to follow because the discontinuous component of acceleration is outside the family of sample paths generated from w_t. This leads to larger than expected errors in tracking and unacceptable errors in prediction. Jinking maneuvers thus achieve precisely the evasive intent of the pilot.

A realistic motion model displays the maneuver acceleration explicitly. Suppose that the maneuver acceleration takes on values in a finite set, say $\{a_1, \ldots, a_s\}$. In the interest of brevity, consider the proposed filter in the context of the engagement described in Ref. 3. The application illustrates many of the issues that arise in tracking agile targets without unduly circumscribing the generality. In summary, suppose that the target moves in the plane with nearly constant speed. It is possible to write the motion equations in a Cartesian coordinate system

$$dx_t = Ax_t\, dt + dw_t + a_tC(k \times v_t^*)\, dt \tag{6}$$

where x_t is a 4-vector containing the vehicle position (components 1 and 2) and velocity v_t (components 3 and 4), v_t^* is a unit vector along v_t, k is a vertical unit vector, and C is a matrix adjusting the dimension of the last factor and linking the maneuver acceleration to the increment in velocity. The first two terms retain their identity from Eq. (1). The final term delineates the maneuver acceleration. It has sense and magnitude given by a_t, and its direction is perpendicular to the velocity. A conventional EKF neglects the last term in Eq. (6) and linearizes the range-bearing measurement about the estimate of x_t.

The maneuver acceleration is best modeled as a jump process (see Ref. 4 and the references therein). Often the power in the wideband accelerations is much smaller than that in the maneuver. A small value of W leads, through P_{xx}, to a long time constant in the EKF. Indeed, the estimate of state generated by Eq. (3) may stray from the target path in certain conditions. An uncompensated bearing-only EKF diverged even when the target did not maneuver.[5] With measurements of both range and bearing, this problem is reduced in the null-maneuver regime for which Eq. (1) is a valid target motion model, but it is far from being eliminated during an acceleration.

A target will maneuver only when it is advantageous to do so. When the amplitudes of the maneuver accelerations are significant, but the frequency low, proper adaptation of LGM techniques is difficult. A small value of P_{xx} leads to good performance in the nonmaneuvering regimes, but response after the initiation of a maneuver tends to be retarded. The conventional way to obviate this difficulty is to simply increase the noise intensity, W; i.e., add "pseudonoise" (see Chap. 9 of Ref. 2). This results in an increase in P_{xx} and has been used to represent a variety of modeling errors. It creates a primitive conservatism in the EKF in so far as the innovations gain is made larger thereby and thus provides a more expeditious response to changes in the acceleration regime. Unfortunately, it also carries with it a needless increase in volatility during quiescent intervals that would be unacceptable in high-accuracy tracking applications where velocity errors are magnified by the predictor.

These difficulties arise because the pseudonoise approach too coarsely approximates the actual behavior of the maneuver process to embed it in the LGM framework. It is far better to use the model given in Eq. (6) directly. It more carefully distinguishes the alternative acceleration hypotheses, and it is reasonable to suppose that the performance of an estimator truly based on it would be superior. Although Eq. (6) does not generate Gaussian sample paths, x_t is conditionally Gaussian given a_t. This fact suggests a multiple hypothesis approach to estimation; i.e., find the conditional expectation of x_t given both Y_t and a_t and then average over the acceleration paths. This technique involves multiple target models with averaging—or mixing—of their outputs. Since there are infinitely many maneuver paths, an extreme pruning of hypotheses must be achieved if the number of active alternatives is not to become overwhelming.[6,7]

In this Note, a path-adaptive estimation algorithm is proposed for tracking an agile target. It is assumed that the conventional center-of-reflection sensor suite is augmented with an imager; i.e., in addition to range-bearing measurements, a sequence of "pictures" of the target is created. A brief study of the advantages that accrue to image enhancement is presented. It is shown that a path-sensitive EKF can be obtained that has good quiescent performance while providing expeditious response to a maneuver. A low filter gain is used in normal operation, thus providing high noise rejection. Alternatively, during transient intervals, the gain is increased to reduce the response time of the filter. The approach is illustrated in the context of a simple example, but it can be used in more general circumstances.

II. Image Augmentation

As indicated in the previous section, center-of-reflection tracking tends to produce unsatisfactory results in volative environments. A measurement that would give a more expeditious indication of a maneuver would reduce the lags inherent in current implementations. In one case, it was proposed that the target transmit a signal to the tracker whenever it turned, thus permitting the estimator to adjust to changing vehicle dynamics.[8] While improved performance was achieved with this novel data link, the referenced application required cooperation between target and tracker. In a hostile environment, the target will select its motion pattern to exploit limitations in the sensor-processor architecture. While increasing W makes more rapid the response to maneuvers, it also degrades quiescent performance. Bekir proposed in Ref. 9 that the a_t be estimated and that the elements of W matrix be augmented proportionately "to the amount of acceleration along each axis." In this way, a "large or unnecessary" increase in W is avoided during nominal operation. Williams and Friedland also addressed this issue and correctly pointed out that the covariance of the acceleration estimation filter should be adjusted "to reflect the increased uncertainty ... due to the transition."[10]

The algorithm presented here preserves the maneuver estimation link described in Ref. 3 but bases enlargement of W on the uncertainty in acceleration instead of its magnitude. Maneuver acceleration is inferred from target rotation as manifested in changes in target orientation in the image.[11] In Ref. 3 the estimated value of the acceleration was added to a simplified—nonmaneuvering—EKF for estimation and prediction. Large peak errors during a maneuver were reduced thereby, and the likelihood of loss of lock lessened. This image-augmented filter has an anomalous property. When the maneuver ends, the estimator is slow to eliminate the sizable residual error and return to nominal operation. This behavior is perhaps surprising because in the nonmaneuvering regime the LGM model correctly describes target motion. The fundamental reason for the slow decay in the error is the failure of the EKF to properly deweight the large residuals generated during the maneuver. The EKF will not perform as expected for an extended period when W is small.

In this Note an estimator is proposed that reduces the response delay found in the filter described in Ref. 3. There is no significant increase in algorithmic complexity over that required in the earlier filter, but instead the information already produced in the image link is exploited in more detail. Specifically, the acceleration uncertainty, which is a byproduct of the output of the image postprocessor, is used to generate appropriate adjustments to the error covariance calculation in the EKF. This simple adjustment is seen to result in significantly faster estimator response.

Consider the proposed filter in the context of tracking a single turn maneuver. Suppose that the wideband acceleration is of low intensity, $(\mathrm{d}w)_i^2 = 0.01\ (\mathrm{m/s^2})^2\,\mathrm{d}t$, and that there is a range-bearing sensor located at the origin of the state space with sample frequency 10 measurements/s and standard errors 5 m in range and 0.25 deg in bearing, independent in time and type. By neglecting a_t in Eq. (6), an EKF can be synthesized using established methods. This provides the basic nonimaging estimate of target location. To determine the usefulness of image augmentation, suppose that there is a collocated imaging-sensor-processor that generates measurements of target orientation. The image processor places the target in one of $L = 16$ (22.5 deg) equally spaced angular bins at the same 10 frames/s rate. Image creation and interpretation is subject to error. It will be assumed that if the true target orientation would place it in bin i, the following errors occur at the following rates (see Ref. 3 for more detail): 1) uniformly distributed errors —1.0%, 2) adjacent bin errors—1.8%, and 3) silhouette projection errors—10.0%

Let Z_t be the filtration generated from the image link, and let $\hat{\alpha}_t$ be the vector of conditional probabilities of individual maneuver accelerations. It is shown in Ref. 3 that $\hat{\alpha}_t$ satisfies the nonlinear stochastic differential equation:

$$\hat{\alpha}_t = (I_s \otimes \mathbf{1}_L')\hat{\phi}_t \tag{7}$$

subject to

$$\mathrm{d}\hat{\phi}_t = Q'\hat{\phi}_t\,\mathrm{d}t + [\mathrm{diag}(\hat{\phi}_t) - \hat{\phi}_t\hat{\phi}_t'](\mathbf{1}_K'\otimes I_L)\Lambda'\mathrm{diag} \times [\Lambda(\mathbf{1}_k'\otimes I_L')\hat{\phi}_t]^{-1}\mathrm{d}\sigma_t \tag{8}$$

where Q is dependent on the manuever and orientation dynamics, Λ is dependent on the frame rate and sensor-image processor precision, I is the identity matrix, $\mathbf{1}$ is a vector of ones, and σ_{ti} counts the number of occurrences of the ith image symbol. The augmented EKF ($\mathrm{EKF_A}$) presented in Ref. 3 simply adds the mean maneuver acceleration $\hat{a}_t$ ($\hat{a}_t = \Sigma_i a_i \hat{\alpha}_t$) to the null EKF.

Figure 1 shows the performance of the augmented tracker on a simple planar maneuver. As expected, the EKF fails to respond expeditiously to the maneuver, and if W were smaller, it would diverge. Tracking is much improved with $\mathrm{EKF_A}$, particularly during the turn, but large errors remain in both filters after the maneuver ends. This is because of the long time constant of both filters resulting from the small value of W.

Note that $\mathrm{EKF_A}$ uses only part of the information on the maneuver generated by the image link, the mean acceleration. With the conditional distribution already available from Eq. (7), it is possible to compute the acceleration variance as well:

$$\mathrm{Var}\ a_t = E\{a_t^2|Z_t\} - (E\{a_t^2|Z_t\})^2 = \Sigma_i a_i^2 \hat{\alpha}_i - (\Sigma_i a_i \hat{\alpha}_i)^2 \tag{9}$$

Equation (9) has an interesting interpretation. When the maneuver is resolvable from the image sequence ($\hat{\alpha}_i \cong e_i$), the acceleration uncertainty is small. Alternatively, as the uncertainty in the maneuver classification increases, Var a_t grows as well. This is just the sort of situational adaptation required in the application. The process noise intensity W can be augmented proportionally to acceleration uncertainty (rather than acceleration magnitude), transformed into the Cartesian coordinate system. The calculated error covariance is the solution to Eq. (4) with the increased W. Through this simple artifice, a maneuver-adaptive tracker is created. Denote the corresponding filter by $\mathrm{EKF_P}$.

Tracking is much improved with this modification. Covariance augmentation improves the speed of filter response and reduces the decay time when the maneuver ends. Figure 1 shows the tracking performance of $\mathrm{EKF_P}$ in position. The velocity estimate is also improved as more detailed information in the image-based link is used. Accurate velocity estimates are of great importance in prediction, and the effect of artifacts in the residuals tends to accumulate in velocity because there is no direct measurement to disabuse the filter of

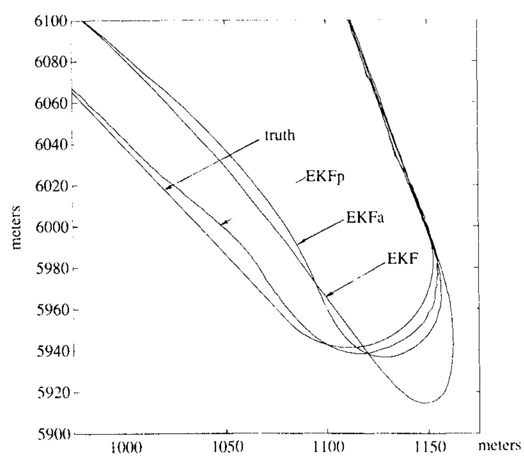

Fig. 1 Tracking performance of the filters.

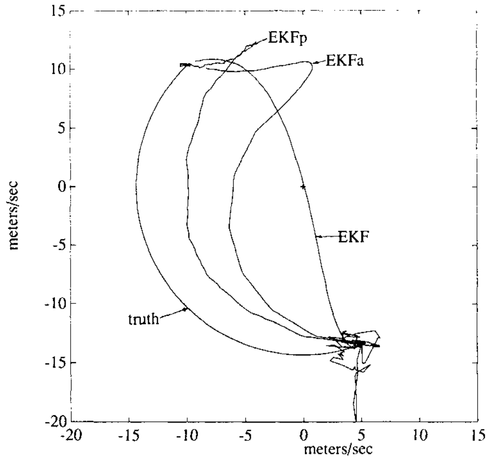

Fig. 2 Velocity performance of the filters.

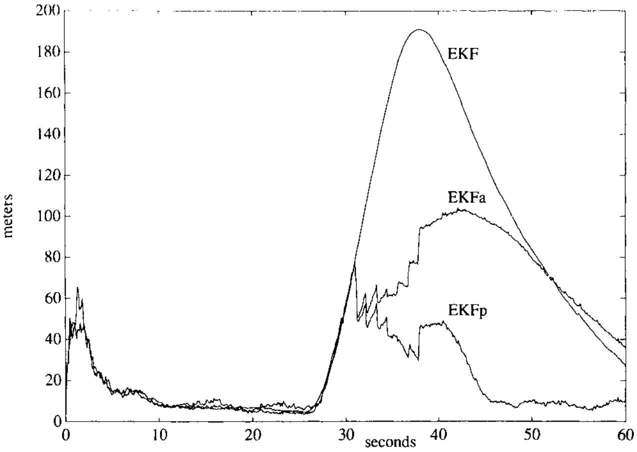

Fig. 3 Prediction performance of the filters.

incorrect references. Figure 2 shows the filter responses in the velocity plane. The EKF is notably deficient in velocity estimation, actually confusing the sense of rotation. As more of the image-derived information is incorporated into the estimate, performance improves. Covariance adaptation particularly speeds the final velocity correction after the maneuver ends.

The improved velocity estimates are reflected in prediction accuracy. Figure 3 shows the prediction responses for the same trajectory. Image-based algorithms have a noticeable local volatility due to the quantization of the orientation grid induced by the image processor. Nevertheless, it is evident that covariance adaptation speeds response. For this scenario the peak 5-s prediction error in position is 190 m, with 25 m remaining 30 s after the maneuver ends. The EKF_A reduces the peak to 110 m but has about the same residual error after 30 s. The adaptive EKF_P has a 78-m peak but only an 8-m prediction error after 15 s.

There is, of course, a penalty to be paid for the improved performance during and after a maneuver. The basic EKF was derived on the basis of a model best tuned to the non-maneuvering target. This null EKF wastes no effort on tracking putative maneuvers, and this complacency leads to excellent nominal prediction performance, approximately a 4-m position error in quiescent null-maneuver sojourns. The adaptive filter "sees" more maneuvers than actually occur. Nevertheless, the performance deterioration is slight, in the neighborhood of 5.5 m during the same conditions. Furthermore, during an acceleration transient, prediction is so improved that the suggested modification will be useful in volative encounters.

III. Conclusions

A covariance adaptive modification of an earlier image-based tracking algorithm improves performance by increasing the filter gain during periods of uncertainty regarding the maneuver regime of the target. Since the increase in algorithmic complexity is slight, this algorithm provides a useful complement to the earlier tracker.

Acknowledgments

This research was supported by a grant from the Hughes Aircraft Co. and by the MICRO Program of the State of California under Project No. 90-158.

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Coupling of Tether Lateral Vibration and Subsatellite Attitude Motion

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Introduction

It has long been known that the study of the motion of tethered systems in space is an intriguing problem. To be more specific, at present, after several years of effort, it seems that not every aspect of the dynamics of TSS-1 (Tethered Satellite System—Mission 1) during retrieval is adequately understood, so that, recently, actions have been taken in the United States and in Italy to increase the control capability of the system. The main simulation difficulties have been pointed out in Ref. 1, where a mixed approach, i.e., one based on

Received Jan. 3, 1991; revision received July 17, 1991; accepted for publication July 18, 1991.

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